Phase transition in computational complex systems

Saravanan Rajendran[1]

June 15, 2016

What is phase transition?

- When a state of a system is evolving to a another sate with the change in any parameter associated with the system is called phase transition.
- For eg. Solid liquid gas transitions, superconducting phase transitions, magnetic transitions

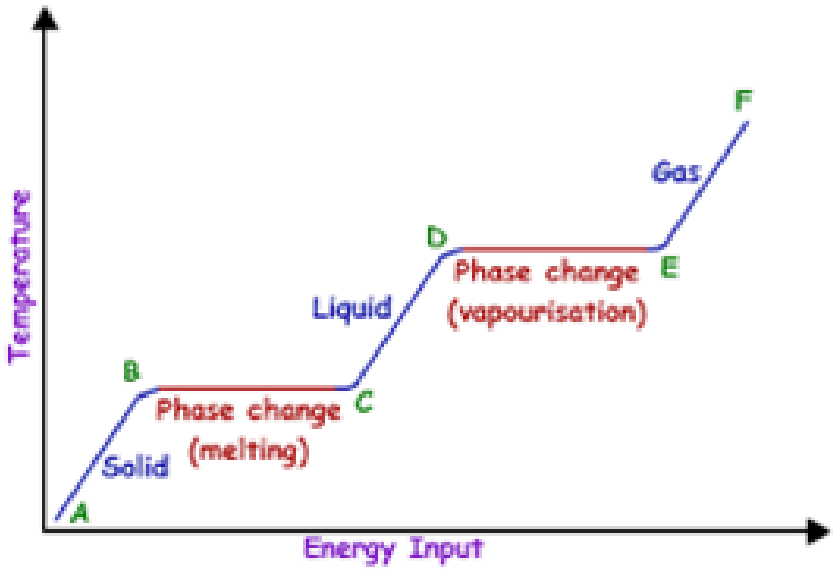


Figure: Phase Transitions

Computational complexity

- The computational algorithms run with the time as a function of its number of variables.
- If an algorithm is a function of polynomial of n (variables) they are said to take polynomial timing and are called P-problems (n, n^2, n^3 etc.,)
- If they are said to take exponential timing ($e^n, 2^n$ etc.,) then the problem is not solved (As computer will be exhausted running throughout the lifetime).

Problem of years!

- In computational world, there are some problems which are unsolved for many years and none ever proved that there exists no solution.
- Those problems (for which P-algorithms neither exist nor proved not exists) are called NP-problems.
- Some of the problems have a characteristic that if a P-algorithm is found to exist for the problem, then it may be employed to all NP-problems, hence called NP-complete problems. (For eg., SAT-Problem[1], travelling salesman problem, graph coloring etc.,)

1

[1] Cook- Levin Theorem

Satisfiability Problem

- The conditions imposed on a problem are solved such that they satisfy all conditions simultaneously.
- This can be solved using boolean algebra or backtrack algorithm.

For eg.,

$$(p' + q).(q + r).(r' + p')$$

Travelling salesman problem

- This problem is finding the minimal length tour for travelling given n number of cities.

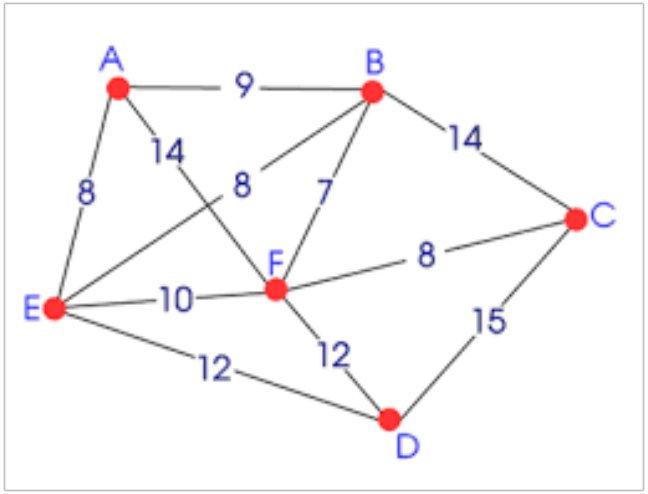


Figure: An example of TSP

- In this problem we focus on the decision problem (does a minimal length tour exists or not?) not closely on the optimization problem(what is the minimal tour?)

Graph coloring

- In graph theory,graph coloring is a special case of graph labeling.
- The vertices of a graph is colored such that no two adjacentvertices or edges share the same color.

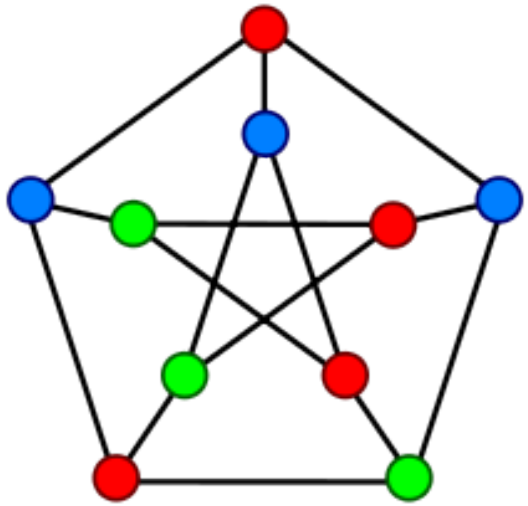
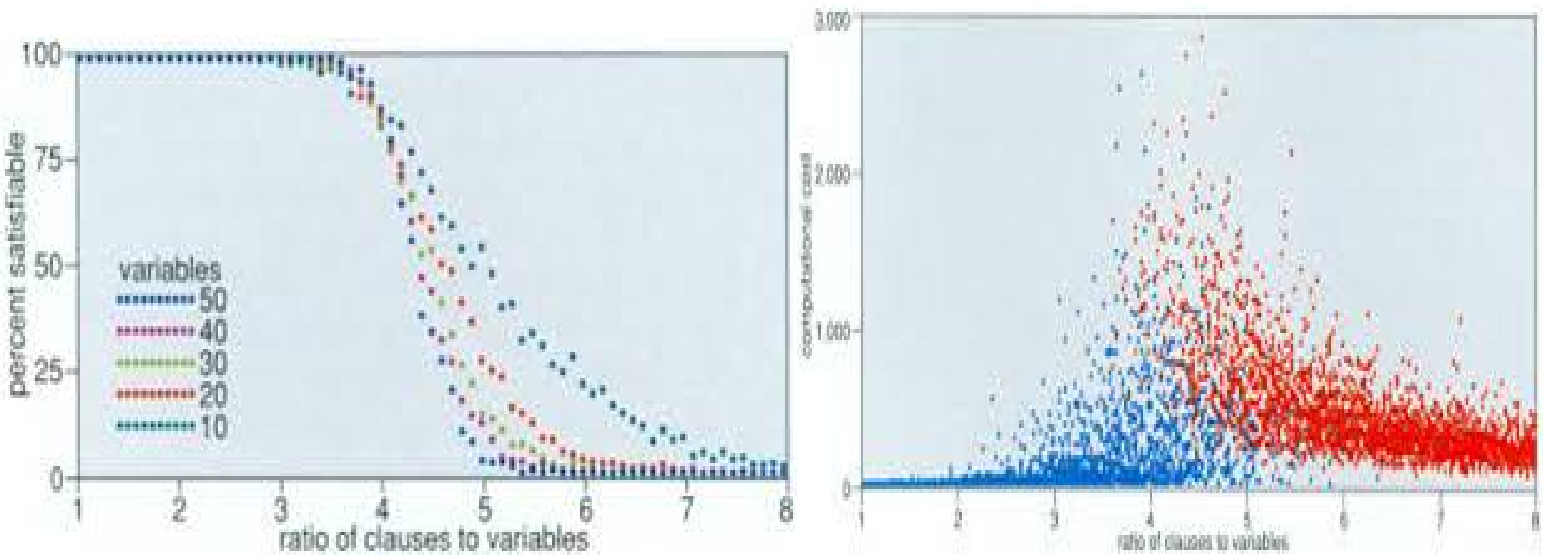


Figure: Petersen graph

Phase transitions!

- As the characteristics of the problem resembles the phase transition (to eyes of an physicist!) we skip the algorithmic approach and study the characteristics of it to solve the problem.



Where really hard problems are!

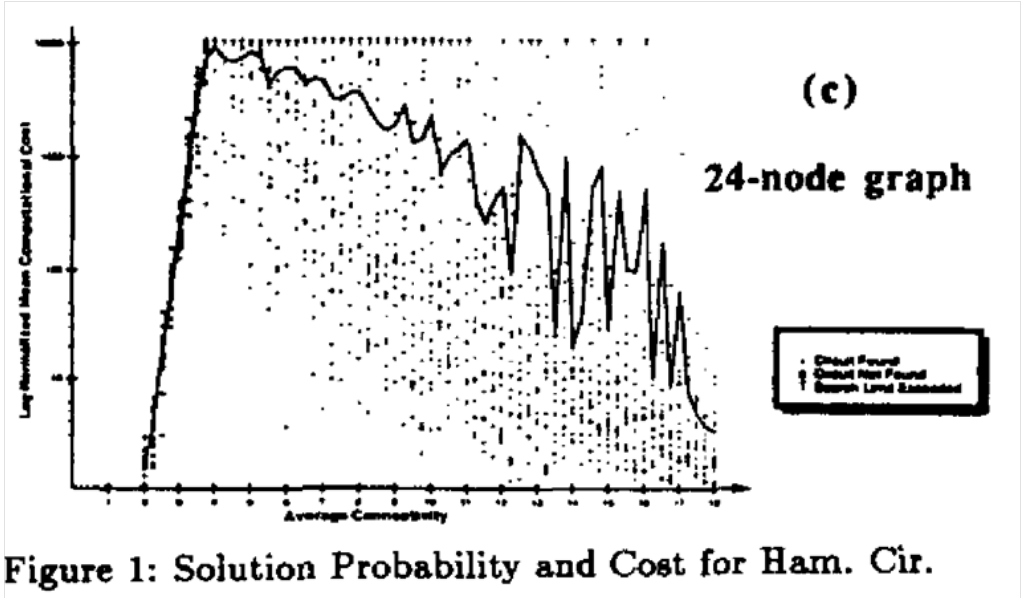


Figure 1: Solution Probability and Cost for Ham. Cir.

Figure: Phase transitions between easy and hard Instances in a hamilton cycle problem

Satisfiability Problem

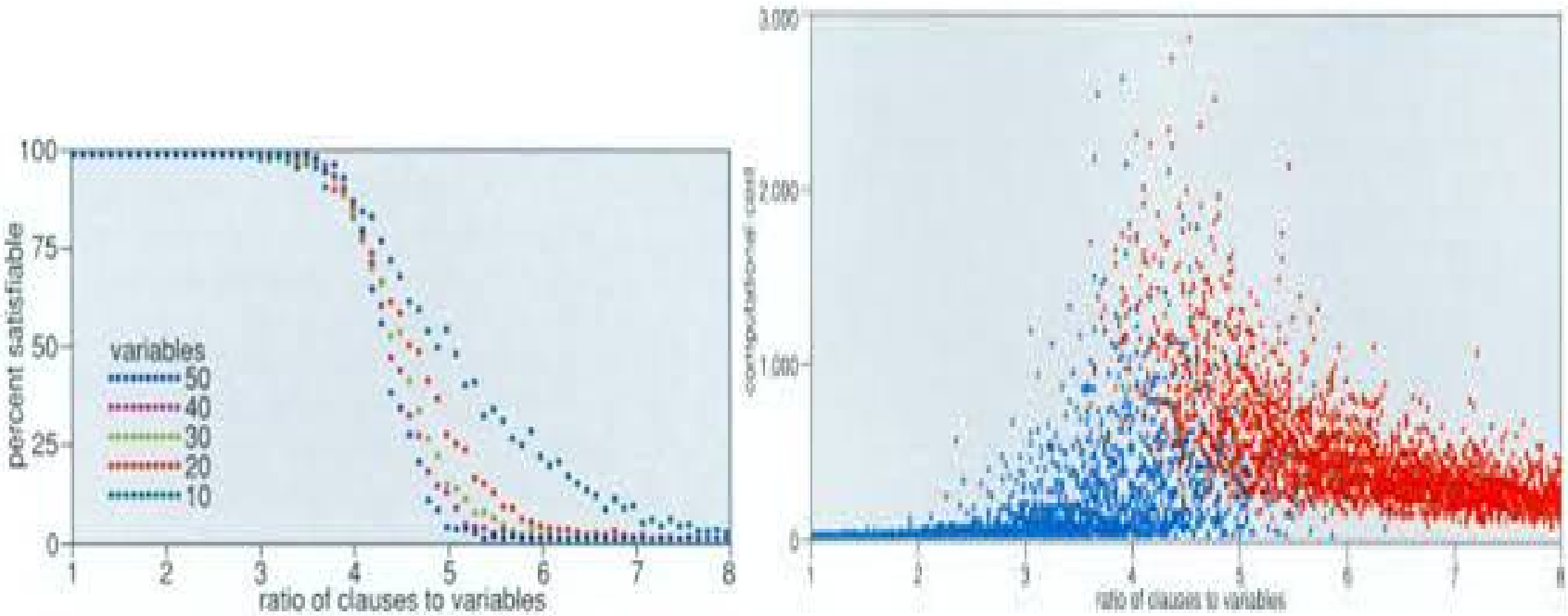
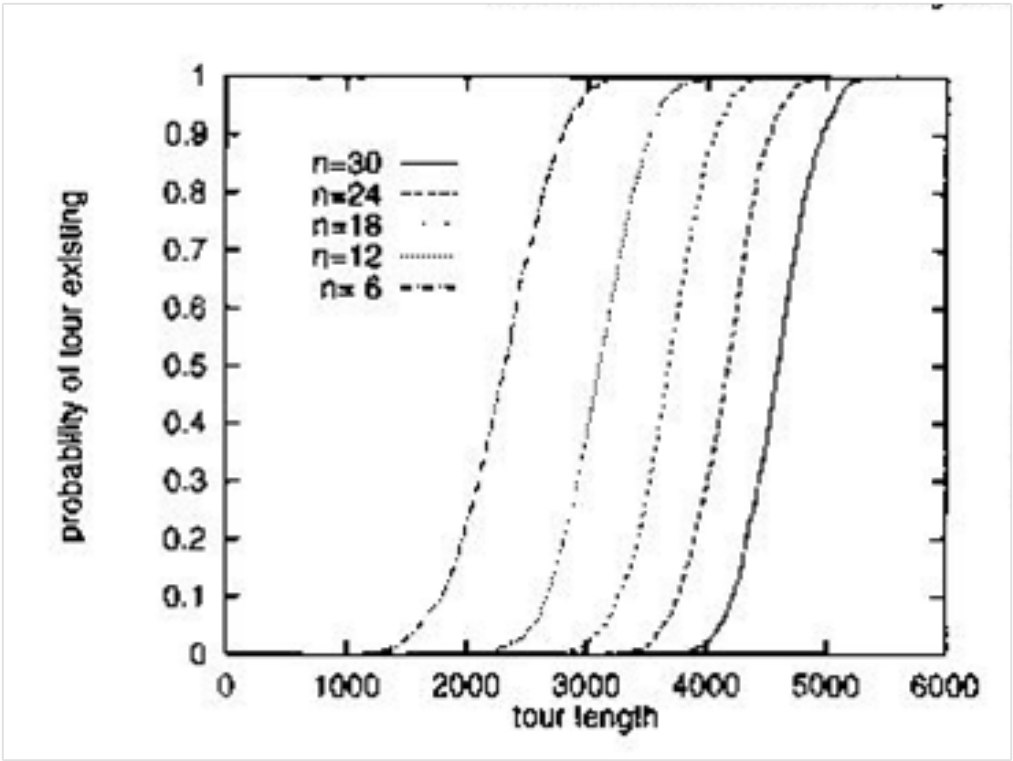


Figure: The figures shows the phase transition in the SAT Problem

Travelling salesman Problem



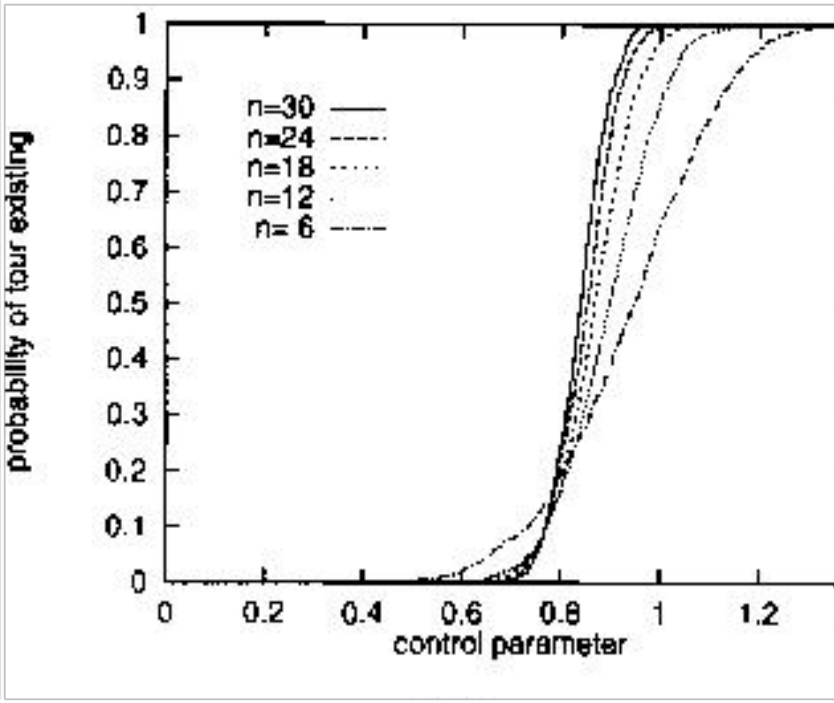
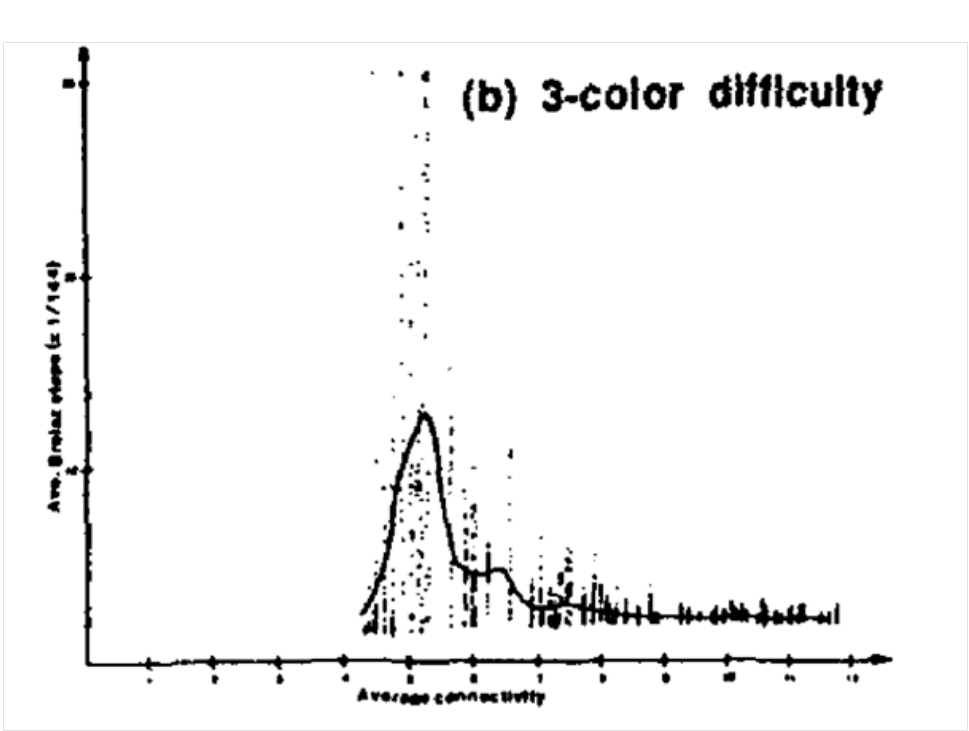


Figure: The figures shows the phase transition of the TSP Problem

Graph coloring



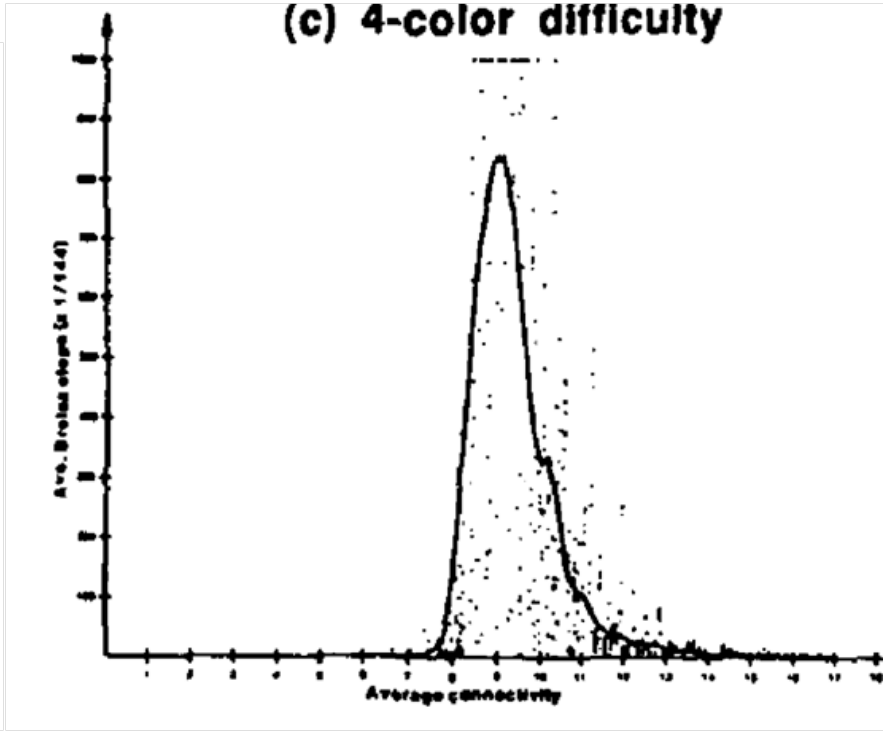


Figure: The figures shows the phase transition of graph coloring

Summary

- Phase transitions occur in all the NP-complete problems
- Studying the characteristics of phase transition in SAT-problem it has been proved that (2+p)-SAT problem has hard instance at p=0.4
- All NP-complete problems have at least one order parameter and the hard to solve problems are around a critical value of this parameter.
- The critical value separates the over constrained and underconstrained regions of the problem space.

References I

Cheeseman, Peter, Bob Kanefsky, and William M. Taylor..
"Where the Really Hard Problems Are."
IJCAI. Vol. 91. 1991.

Gent, Ian P., and Toby Walsh.
"The TSP phase transition."
Artificial Intelligence 88.1 (1996): 349-358.

Hayes, Brian.
"Computing Science: Can't get no satisfaction."
American scientist 85.2 (1997): 108-112.

Thank You!